

Quels degrés de liberté pour quels phénomènes?  
Part II.  
La coexistence de formes par les méthodes  
au delà du champ moyen

Michael Bender

Institut de Physique Nucléaire de Lyon, CNRS/IN2P3, Université de Lyon, Université Lyon 1  
69622 Villeurbanne, France

Journées SFP–BTPN sur  
les grandes questions en physique nucléaire fondamentale  
Campus Michel-Ange Paris-16  
21-22 Juin 2016



Université Claude Bernard



# Which are the irreducible ingredients of a (minimal) predictive model of shape coexistence and its experimental signatures? ?

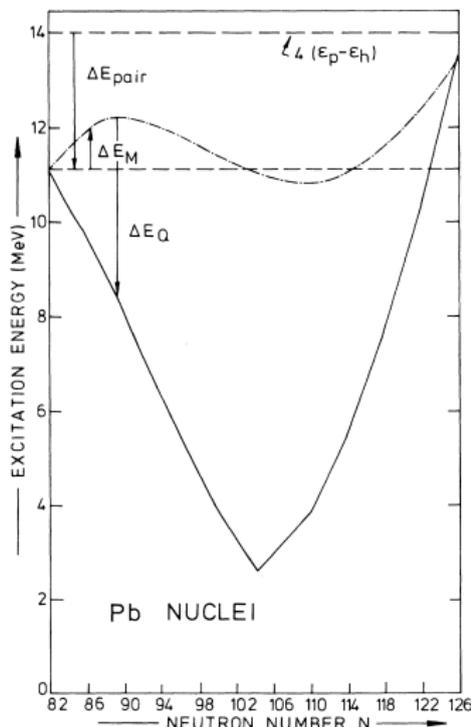
- ▶ What is there to be modeled?
  - ▶ sequence of levels and their excitation energies
  - ▶  $E0$  transition matrix elements
  - ▶  $E2$  transition matrix elements
  - ▶ (charge) radii (and isotopic shifts)
  - ▶ masses (and mass differences)
- ▶ Distinguish
  - ▶ deformation softness (states spread over a wide range of deformations)
  - ▶ shape coexistence (distinguishable states that might be directly mixed)
  - ▶ shape entanglement (distinguishable states that can only be mixed via third states) Poves, JPG 43 (2016) 020410.
- ▶ role of  $np$ - $n$  hole excitations involving intruder / extruder states?

# Which are the irreducible ingredients of a (minimal) predictive model of shape coexistence and its experimental signatures? ?

- ▶ What is there to be modeled?
  - ▶ sequence of levels and their excitation energies
  - ▶  $E0$  transition matrix elements
  - ▶  $E2$  transition matrix elements
  - ▶ (charge) radii (and isotopic shifts)
  - ▶ masses (and mass differences)
- ▶ Distinguish
  - ▶ deformation softness (states spread over a wide range of deformations)
  - ▶ shape coexistence (distinguishable states that might be directly mixed)
  - ▶ shape entanglement (distinguishable states that can only be mixed via third states) Poves, JPG 43 (2016) 020410.
- ▶ role of  $np$ - $n$  hole excitations involving intruder / extruder states?
- ▶ Early *ad-hoc* model of shape coexistence: estimate excitation energy of  $0^+$  states from the difference in (spherical) single-particle energies, the change in pairing energy, a monopole correction and the quadrupole correlation energy.

Heyde *et al*, PRC44 (1991) 2216

Heyde & Woods, RMP 83 (2011) 1467

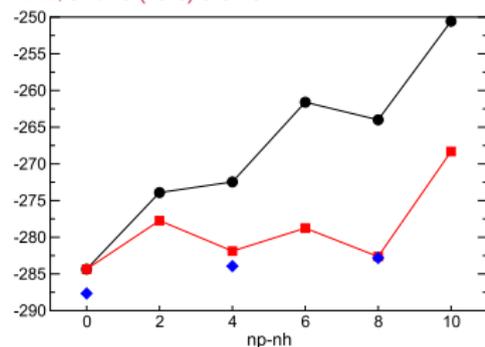


- ▶ Shell model: [Poves, JPG 43 \(2016\) 020410](#).
  - shape remains implicit
  - + good quantum numbers
  - + band mixing
  - intruder states require two major shells
- ▶ Interacting boson model: [Nomura et al JPG 43 \(2016\) 020408](#).
  - shape remains implicit
  - + good quantum numbers
  - + band mixing
- ▶ (Self-consistent) mean field:
  - + energy surfaces with multiple minima
  - no quantum numbers, nor selection rules
  - non-orthogonal states
  - no mixing of bands
- ▶ "beyond mean field" by projected GCM:
  - + projection → quantum numbers & selection rules
  - + Generator Coordinate Method → band mixing
  - computationally intensive
- ▶ "beyond mean field" with collective Hamiltonians
  - + quantum numbers & selection rules
  - + band mixing

# State-of-the-art modeling of shape coexistence

- ▶ **Shell model:** Poves, JPG 43 (2016) 020410.
  - shape remains implicit
  - + good quantum numbers
  - + band mixing
  - intruder states require two major shells
- ▶ **Interacting boson model:** Nomura et al JPG 43 (2016) 020408.
  - shape remains implicit
  - + good quantum numbers
  - + band mixing
- ▶ **(Self-consistent) mean field:**
  - + energy surfaces with multiple minima
  - no quantum numbers, nor selection rules
  - non-orthogonal states
  - no mixing of bands
- ▶ **"beyond mean field" by projected GCM:**
  - + projection → quantum numbers & selection rules
  - + Generator Coordinate Method → band mixing
  - computationally intensive
- ▶ **"beyond mean field" with collective Hamiltonians**
  - + quantum numbers & selection rules
  - + band mixing

Poves, JPG 43 (2016) 020410.



Shell-model analysis of  $0^+$  levels in  $^{40}\text{Ca}$

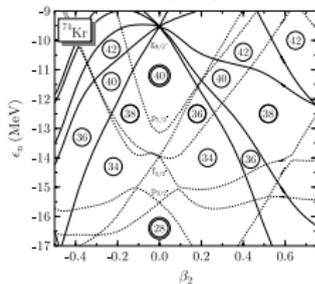
**black:** lowest Slater determinant in given  $np$ - $nh$  subspace

**red:** lowest mixed state in given  $np$ - $nh$  subspace

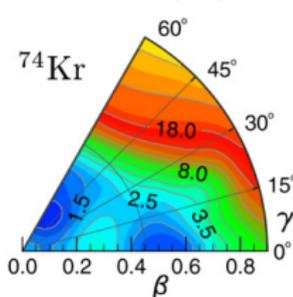
**blue:** full shell model calculation

# Which model ingredients are really relevant?

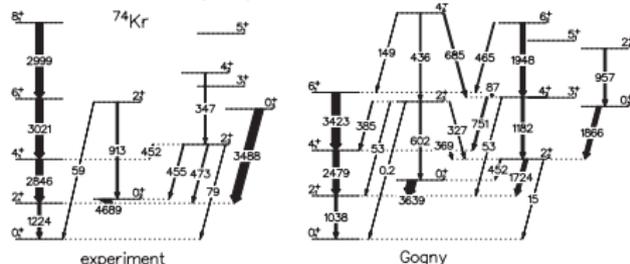
M. B. *et al*, PRC74 (2006) 024312.



Girod *et al*, PLB676 (2009) 39.



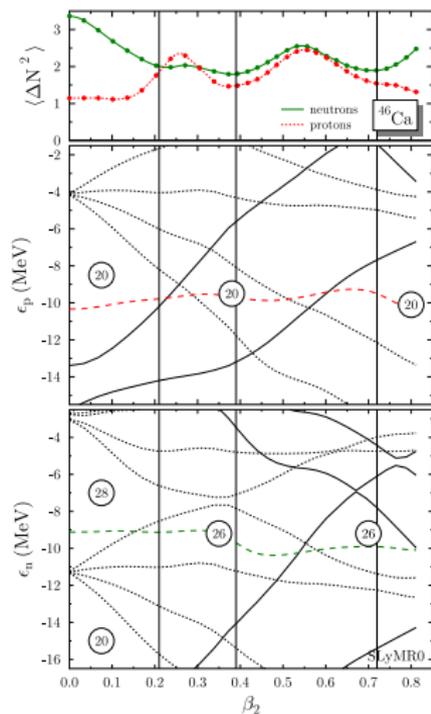
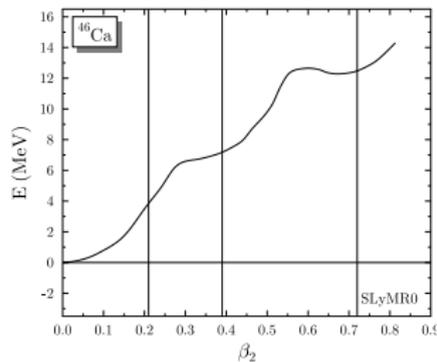
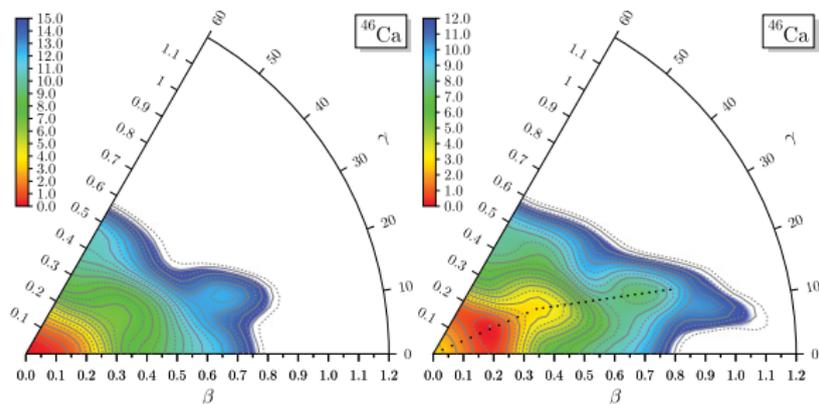
Clément *et al*, PRC75 (2007) 054313.



- ▶ Which are the irreducible ingredients of a (minimal) predictive microscopic model of shape coexistence and its experimental signatures?
  - ▶ quantum mechanics
  - ▶ shell structure and distinguishable configurations (that have different shape or that can be associated with different shapes)
  - ▶ different mean fields (RPA-type methods fail for shape coexistence)
  - ▶ collectivity
  - ▶ configuration mixing (orthogonality, band mixing, ...)
  - ▶ good quantum numbers (for selection rules of transitions).
  
- ▶ Is there an *effective field theory* of shape coexistence?

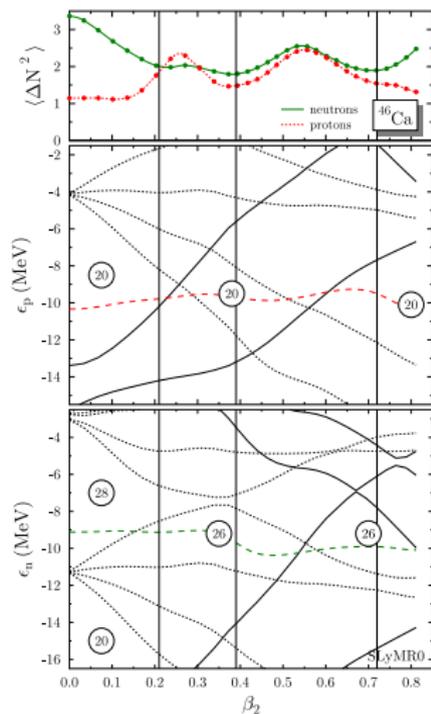
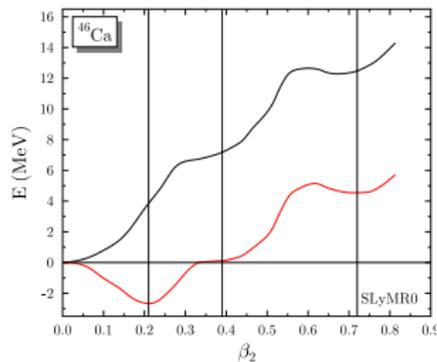
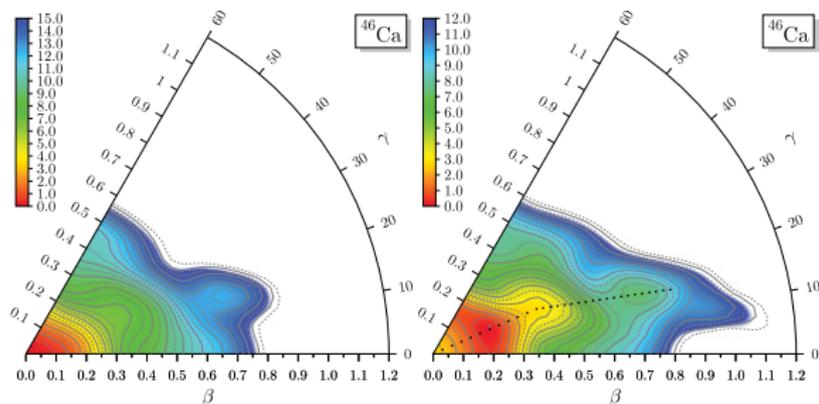
For recent work toward an effective field theory of collectively rotating and/or vibrating deformed nuclei see Papanbrock *et al*, NPA852 (2011) 36; Zhang *et al*, PRC87 (2013) 034323; Coello-Pérez *et al*, PRC92 (2015) 014323

# Shell-model interpretation of beyond-mean-field states and vice versa



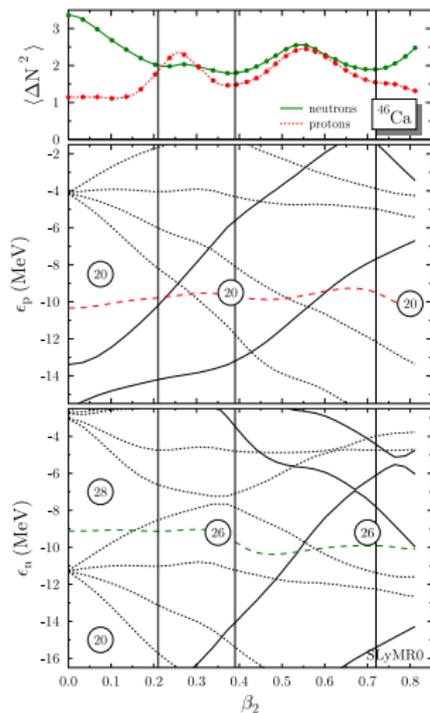
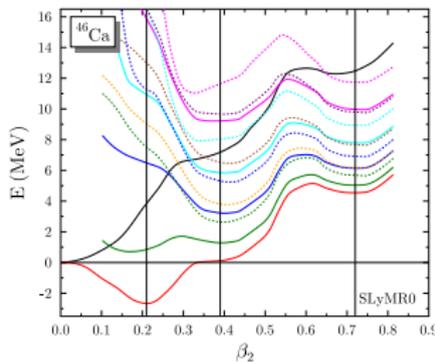
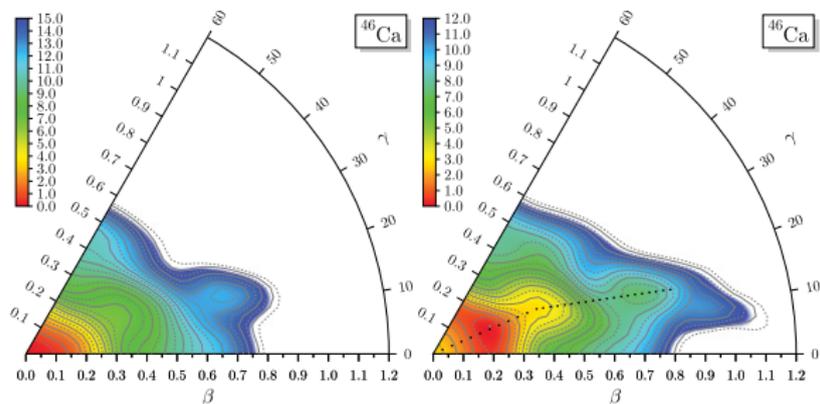
M. B., B. Bally, P.-H. Heenen, unpublished

# Shell-model interpretation of beyond-mean-field states and vice versa



M. B., B. Bally, P.-H. Heenen, unpublished

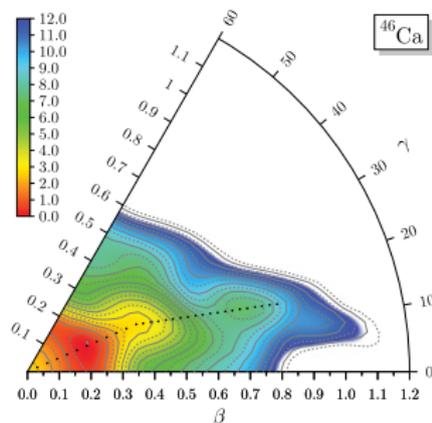
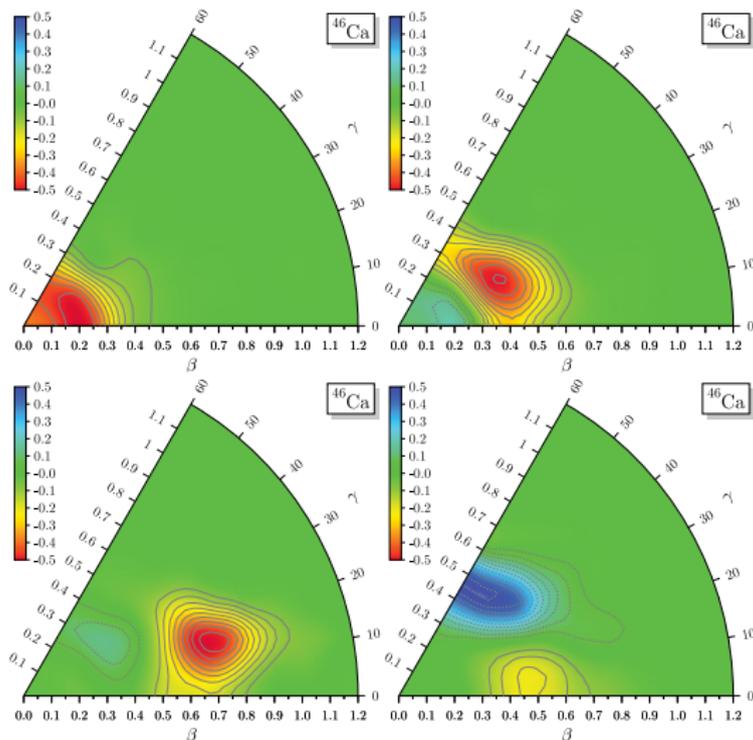
# Shell-model interpretation of beyond-mean-field states and vice versa



M. B., B. Bally, P.-H. Heenen, unpublished

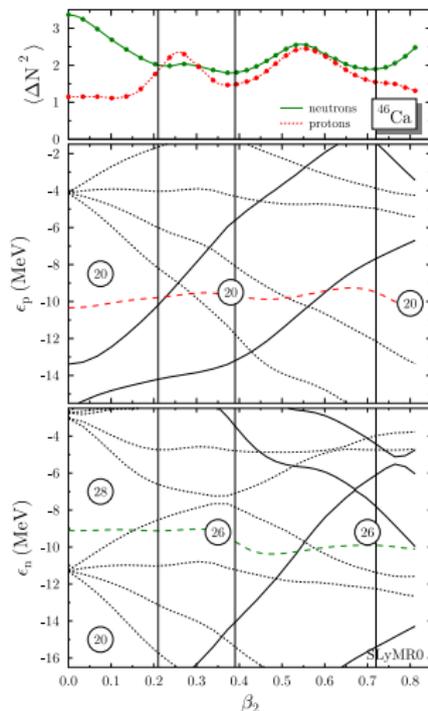
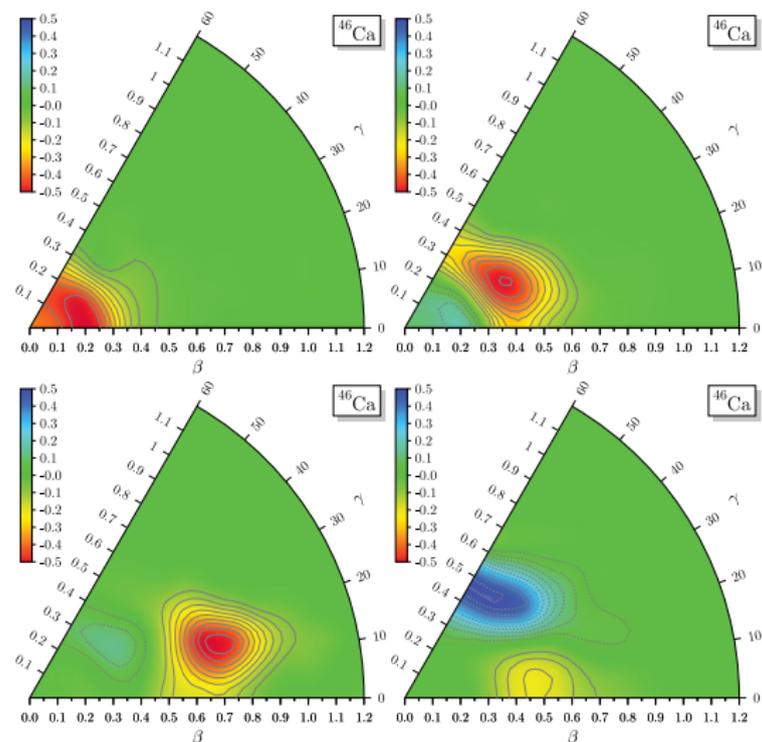
# Shell-model interpretation of beyond-mean-field states and vice versa

collective wave function of the four lowest  $0^+$  states



M. B., B. Bally, P.-H. Heenen, unpublished

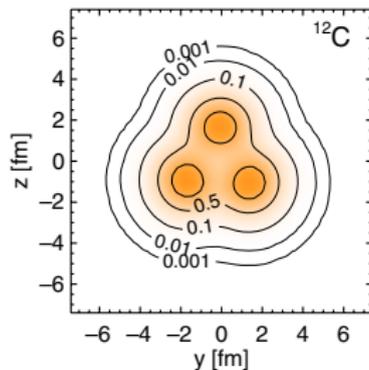
collective wave function of the four lowest  $0^+$  states



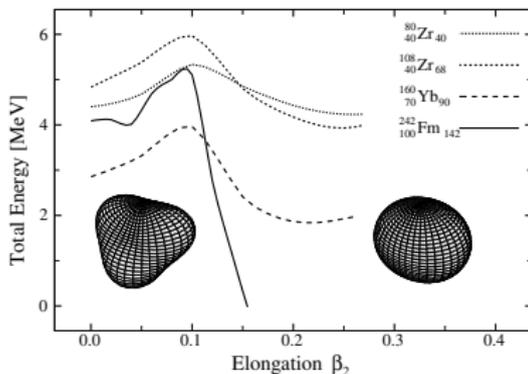
M. B., B. Bally, P.-H. Heenen, unpublished

- ▶ All examples shown so far concern the coexistence of shapes with different quadrupole moment.
- ▶ Are there coexistences driven by other shape degrees of freedom?
  - ▶ clustering.
  - ▶ octupole?
  - ▶ hexadecapole?
  - ▶ tetrahedral or octahedral shapes?
- ▶ Are they also driven by  $np$ - $nh$  excitations or something else?

Neff, EPJST156 (2008) 69

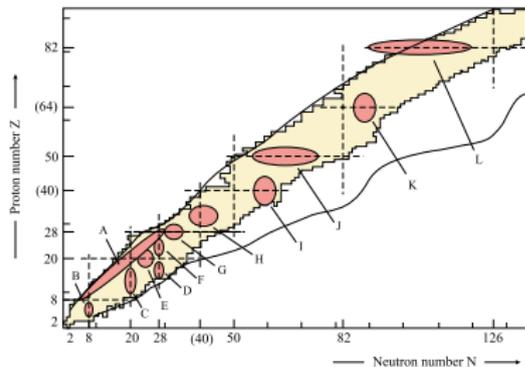


Dudek et al, PRL88 (2002) 252502



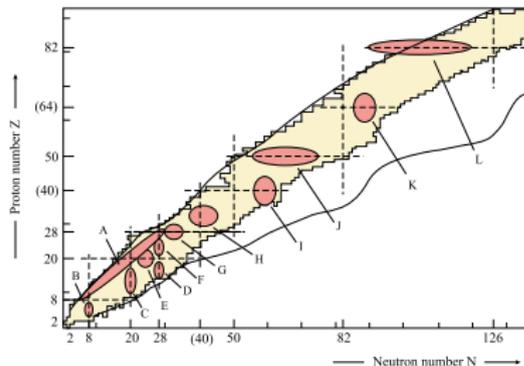
# Coexistence in normal nuclei, exotic nuclei, and elsewhere

Heyde & Woods, RMP 83 (2011) 1467

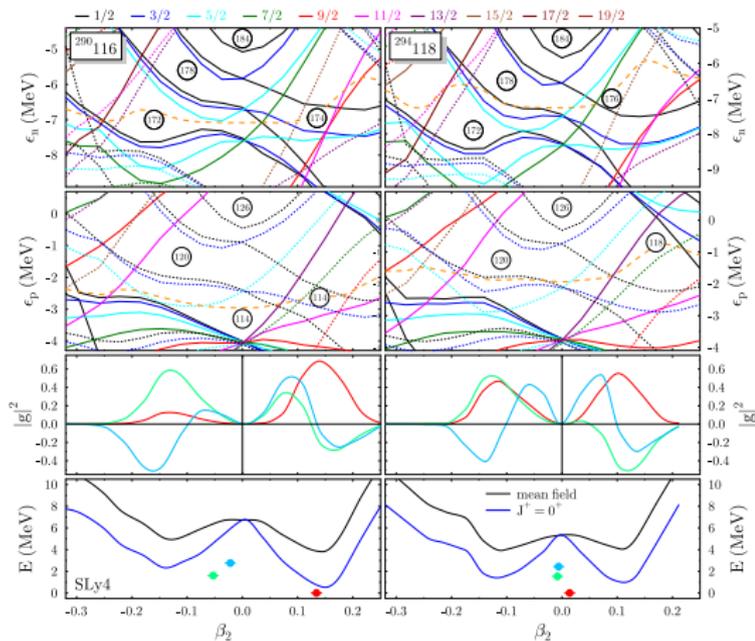


# Coexistence in normal nuclei, exotic nuclei, and elsewhere

Heyde & Woods, RMP 83 (2011) 1467



Heenen, M. B., Bally & Ryssens, to be published



- ▶ Profiting from high-performance computing, over the last few years the range of applicability of the shell model and of beyond-mean-field methods has been enlarged such that both methods begin to cover the physics relevant for shape coexistence (intruder states, good quantum numbers, configuration mixing, ...).
- ▶ Shape coexistence emerges in both methods in similar situations:  $np$ - $nh$  excitations involving intruder states.
- ▶ In the context of the shell these states are usually interpreted "vertically" in terms of occupations of spherical shells ("islands of inversion").
- ▶ In the context of self-consistent mean-field models "and beyond" these states are usually interpreted "horizontally" in terms of gaps in the Nilsson diagram.

- ▶ Profiting from high-performance computing, over the last few years the range of applicability of the shell model and of beyond-mean-field methods has been enlarged such that both methods begin to cover the physics relevant for shape coexistence (intruder states, good quantum numbers, configuration mixing, ...).
- ▶ Shape coexistence emerges in both methods in similar situations:  $np$ - $nh$  excitations involving intruder states.
- ▶ In the context of the shell these states are usually interpreted "vertically" in terms of occupations of spherical shells ("islands of inversion").
- ▶ In the context of self-consistent mean-field models "and beyond" these states are usually interpreted "horizontally" in terms of gaps in the Nilsson diagram.

The difference in interpretation appears to be more "cultural" than physical.

## What is Shape Coexistence?

*"Shape coexistence is a very peculiar nuclear phenomenon consisting in the presence in the same nuclei, at low excitation energy, and within a very narrow energy range, of two or more states (or bands of states) which: (a) have well defined and distinct properties, and, (b) which can be interpreted in terms of different intrinsic shapes."*

A. Poves, foreword to the 2015 special issue of JPG on "Shape coexistence in nuclei"

- ▶ Shape coexistence is a generic feature of atomic nuclei that in one way or the other is exhibited by the majority of nuclei. It can come in many flavours:
  - ▶ coexisting structures in regions of transitional nuclei (evolution with shapes with filling of shells)
  - ▶ island(s) of inversion
  - ▶ rotational bands of "spherical nuclei" including doubly-magic ones ( $^{16}\text{O}$ ,  $^{40}\text{Ca}$ ,  $^{56}\text{Ni}$ , ...)
  - ▶ fission isomers / superdeformation / hyperdeformation
  - ▶ clustering
- ▶ Shape coexistence imprints its presence on (the systematics of) virtually all spectroscopic properties of nuclei at low excitation energy.